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A PROCESS FOR THE DIMENSIONING OF THE HIGH EFFICIENCY PLATE FINS OF COMPACT HEAT EXCHANGERS

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Abstract—It is pointed out that the non-uniform temperature changes of a medium, flowing along plate fins, resulting from different temperature changes taking place adjacent to the fin base and at the fin tip and the heat conductance of the fin in the direction of flow, must not be neglected when dimensioning the finned surface.

It is shown that with small mass rates of flow, with efficient fin design, or with long fins in the direction of flow, as for instance with compact fins having very good heat-transfer coefficients, tubes finned longitudinally in an axial direction, etc., such neglect may lead to errors of 10–30 per cent.

Having evaluated the final results of their investigations by computer, [2–4], the authors present design charts in which these effects have been taken into account.

The authors finally describe a procedure, based on their charts, which enables the results of measurements on finned surfaces to be evaluated more simply, quickly and accurately than with the usual fin efficiency formula which neglects the above effects.

NOMENCLATURE

h_x, fin dimension normal to flow;

h_y, fin dimension in flow direction;

m_x, $m_x = \sqrt{\dfrac{2\alpha}{\lambda_x v_0}}$;

r, value characteristic of the fin design— $r = \dfrac{I}{F_0}\dfrac{2h_x^2}{\lambda_x v_0}$;

v_0, fin thickness;

x, y, coordinates of place; x = normal to flow, y in flow direction;

A_u, A_v, dimensionless numbers— $A_u = \dfrac{\lambda_x}{2\alpha}\dfrac{v_0}{h_x^2}$; $A_v = \dfrac{\lambda_y}{2\alpha}\dfrac{v_0}{h_y^2}$;

C, dimensionless number—$C = \dfrac{C_k}{2\alpha h_y}$;

C_k, part of the thermal capacity of the rate of mass flow related to unit fin length in direction x;

F_0, $F_0 = 2h_x h_y$ the heat-transfer surface of the fin;

I, $I = C_k h_x$ the thermal capacity of the rate of mass flow along the fin;

T_0, fin base temperature;

T_{k0}, temperature of the medium at the inlet;

T_k, temperature of the medium behind the fin;

Δt_f, $\Delta t_f = T_k - T_{k0}$ the warming up of the medium;

ΔT_0, $\Delta T_0 = T_0 - T_{k0}$, the difference between the temperature of the medium at inlet and the fin temperature;

$\Delta T_{\log}$, logarithmic mean temperature difference between fin base and medium;

Q, heat transferred to one fin per unit time;

α, heat-transfer coefficient between medium and fin;

ε, fin efficiency;

ε_L, dimensionless number, similar to fin efficiency $\varepsilon_L = \dfrac{Q}{2h_x h_y \Delta T_0 \alpha}$;

ε_R, approximate value of the fin efficiency;
λ_x, λ_y, fin conductance in x resp. y direction;
ϕ_h, effectiveness of heat exchanging equipments;
ω, $\omega = \frac{2n-1}{2}\pi$ where the "n" index denotes the serial number in the infinite series;
ϕ_k, dimensionless number characterizing the themperature changes in the medium flowing along the fin, fin effectiveness.

THE RAPID development of chemistry and power engineering and the increasingly stringent demand for compact power machines (gas turbines, nuclear drives, etc.) urgently call for high-efficiency compact heat exchangers.

Research work in this field, aims among other things, at the design and construction of high-efficiency heat exchangers with plate fins and laminar flow.

Such heat exchangers are characterized generally by more or less laminar flow of the medium along the plate fins, over a relatively long path, without mixing.

Given the heat-transfer coefficient, which in laminar flow is relatively easy to determine, the heat exchange taking place in plate fins is readily calculable by means of the fin efficiency concept [1]. When computing fin efficiency, it is the phenomena of heat transfer and heat conductance which arise along the fin simultaneously, and mutually determine each other's boundary conditions, that are taken into consideration, but the following factors are neglected:

(1) the heat conductance of the fin in the direction of flow;
(2) the warming up of the medium along the fin, and
(3) the temperature changes in the fin base, in the direction of flow.

These approximations, however, are inadmissible if heat exchange takes place with the medium in laminar flow over a long path without mixing—as mentioned above.

Ad 1. In deriving the relationships to be used in the computation of fin efficiency, it is always assumed that the fin material conducts heat towards the fin base only. But this assumption holds only if the changes in the temperature of the medium flowing along the fin are substantially less than the temperature difference which produces heat conduction in the fin proper. Should, however, the heat-transfer coefficient be very good, the thermal capacity of the flowing medium very small, or the fin length in the direction of flow relatively large—i.e. should the fin be very "deep"—then the temperature of the medium along the fin will undergo a considerable change, and cause a corresponding temperature gradient in the fin to arise, not only in the direction of the base but also in that of the flow.

Ad 2. Another complicating factor is the non-uniformity of the temperature change that takes place in the medium: the fin base will cause a greater change in the temperature of the medium flowing adjacent to it than the fin tip, which has a lesser effect on the temperature pattern, due to the thermal resistance of the fin.

Ad 3. A third change—similarly neglected in the literature—may occur in the value of the fin efficiency on account of the changes in the temperature of the fin base in the flow direction.

It should be pointed out that the above simplification, as will be proved also numerically, is inadmissible if the fins are relatively long and the thermal capacity of the flowing medium is low in comparison with the heat-transfer coefficient. This is often the case, for instance, in tubes with longitudinally arranged fins (which are increasingly applied in modern heat exchangers).

For the accurate dimensioning of heat exchangers of the type outlined above, the authors have in [2–4] elaborated a number of relationships suitable for computing the efficiency of plate fins, taking into consideration also the above-mentioned effects. These relationships were obtained in the form of infinite series.

The correction factor related to the temperature difference as measured on the inlet edge of the fin [similar to fin efficiency—for definition see equation (16) in more detail] can be calculated, at constant fin base temperature in the following manner:

$$\frac{\varepsilon_L}{C} = 1 - \sum_{n=1}^{\infty} \frac{2e^{\varepsilon_{3n}}}{\omega^2} \frac{1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - e^{\varepsilon_{1n}-\varepsilon_{2n}}\left[1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right] + e^{\varepsilon_{1n}-\varepsilon_{3n}}\left[\left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right]}{1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2 - e^{\varepsilon_{1n}-\varepsilon_{2n}}\left[1 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2\right] - e^{\varepsilon_{3n}-\varepsilon_{2n}}\left[\left(\frac{\varepsilon_{3n}}{\varepsilon_{2n}}\right)^2 - \left(\frac{\varepsilon_{3n}}{\varepsilon_{1n}}\right)^2\right]}, \quad (1)$$

where

$$\varepsilon_{1n}, \quad \varepsilon_{2n} \quad \text{and} \quad \varepsilon_{3n}$$

are three roots of an equation of the third degree:

$$S^3 + \frac{1}{C}S^2 - \frac{1 + \omega^2 A_u}{A_v}S - \frac{A_u\omega^2}{A_v C} = 0. \quad (2)$$

It should be noted that the relationship of ε_L to fin efficiency is also specified in the quoted papers, likewise at constant fin base temperature:

$$\varepsilon = C \ln \frac{C}{C - \varepsilon_L}. \quad (3)$$

This paper gives those diagrams for dimensioning which were obtained by the given relationships on a digital computer. The diagrams refer to unchanged base temperature.

The relationship for varying fin base temperature is already available [4]. Its processing by electronic computer is in progress.

CHART PARAMETERS

With a view to satisfying practical demands, the following values have been chosen for the parameters of the charts:

1. $$C = \frac{C_k}{2\alpha h_y}, \quad (4)$$

2. $$\sqrt{\left(\frac{1}{A_u}\right)} = h_x\sqrt{\left(\frac{2\alpha}{\lambda_x v_0}\right)} = m_x h_x \quad (5)$$

of which $m_x h_x$ will appear below in this form: $[mh]_x$ or, more abbreviated, as mh_x where the index x indicates that when calculating the "mh" dimensionless value, both the longitudinal fin dimension and the heat conductance of the fin in direction x must be taken into consideration.

3. $$\sqrt{\left(\frac{A_v}{A_u}\right)} = \left(\sqrt{\frac{\lambda_y v_0}{2\alpha}}\right)\frac{1}{h_y}\left(\sqrt{\frac{2\alpha}{\lambda_x v_0}}\right)h_x = \frac{h_x}{h_y}\sqrt{\left(\frac{\lambda_y}{\lambda_x}\right)}. \quad (6)$$

CALCULATION OF FIN EFFICIENCY WITH SLOTTED RIBS

The so-called slotted-fin heat exchanger is a special construction among high-efficiency heat exchangers. It uses fins heavily slotted in the flow direction for better heat exchange.

Due to the fact that slots perpendicular to the flow, cut, as it were, the heat conductance of the material in the y-direction, thus fulfilling the criterion whereby $\lambda_y = 0$, the value of:

$$\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}},$$

with slotted fins is equal to zero.

DESCRIPTION OF THE ENCLOSED CHARTS

The values arrived at by the computation have been plotted in the charts, Figs. 1–6. On their abscissae the C and on their ordinates the

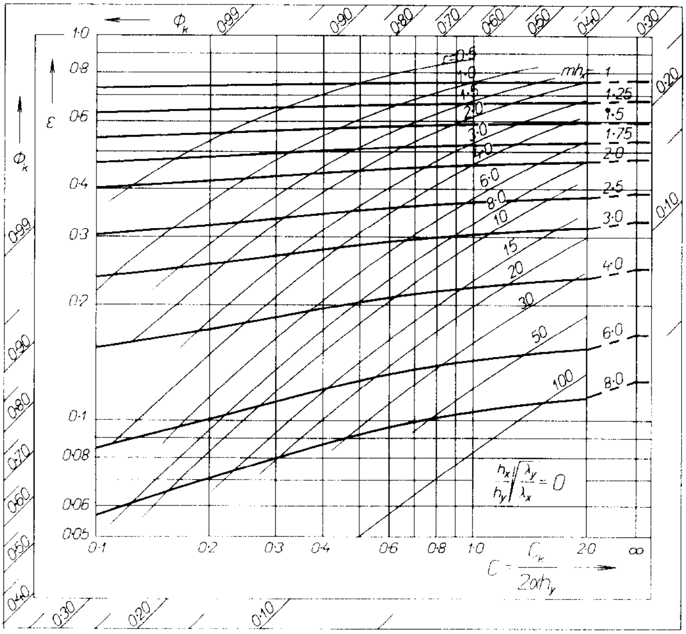

FIG. 1. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

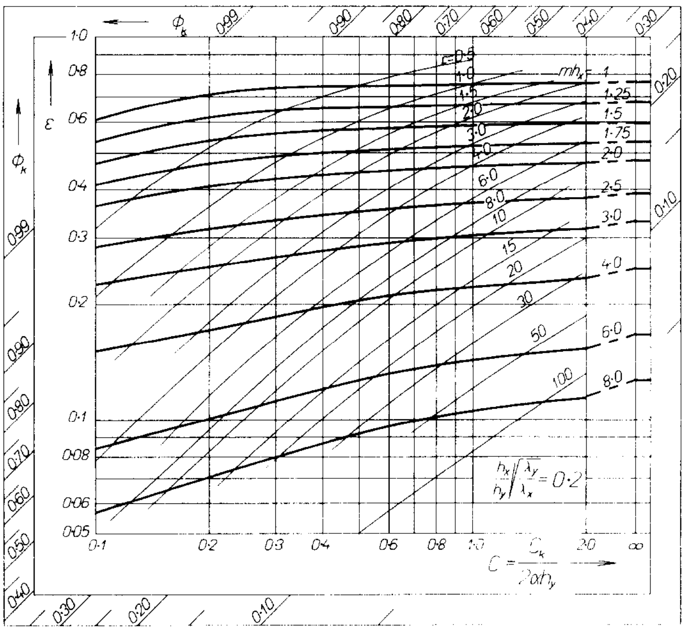

FIG. 2. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

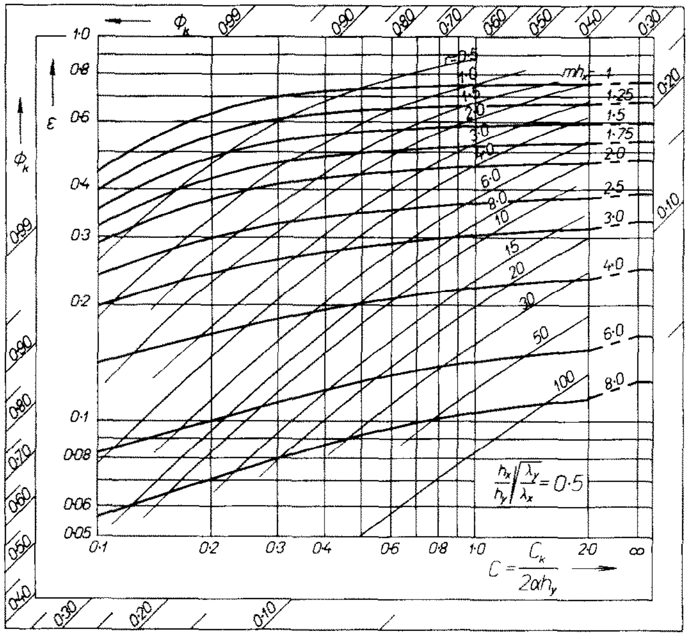

FIG. 3. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

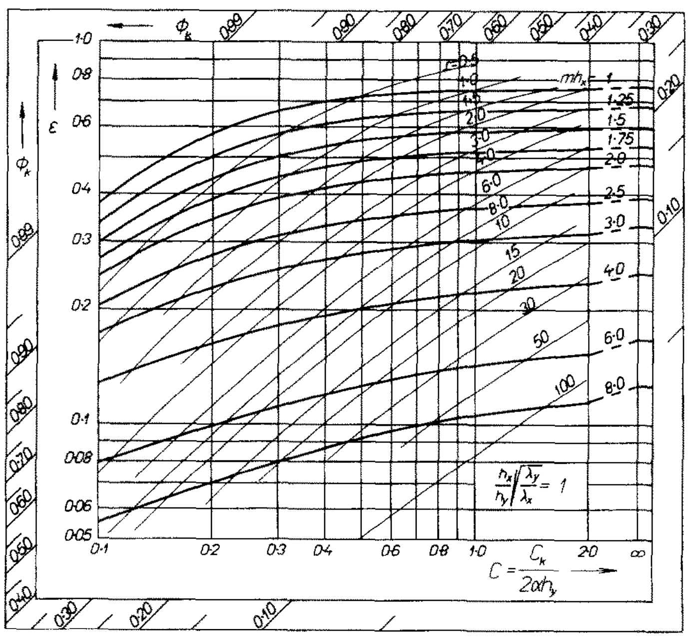

FIG. 4. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

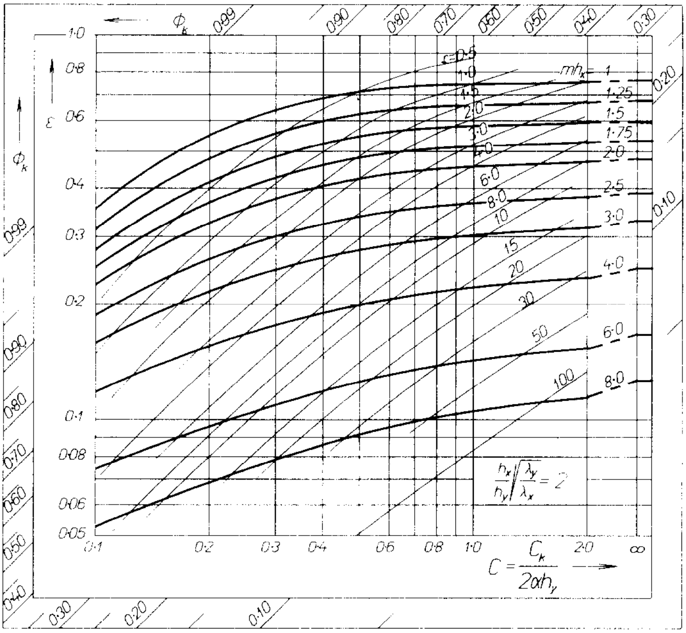

FIG. 5. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

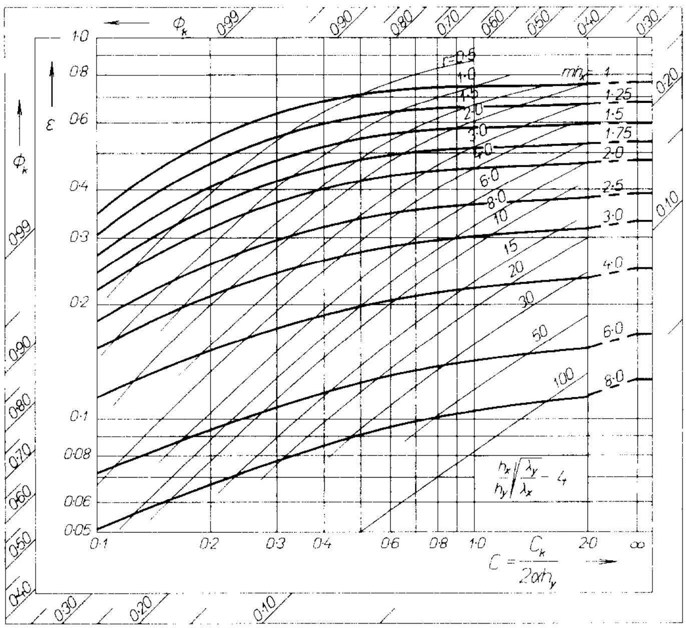

FIG. 6. Accurate plate fin efficiency which takes account of the heat conductance of the fin in flow direction and the non-uniform changes in the temperature of the flow medium.

fin efficiency, ε, have been indicated, while the parameter of the set of curves is the value of mh_x.

The value of

$$\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}$$

is constant within each chart.

Accordingly, the entire area under examination has been encompassed in six charts, while fin efficiencies pertaining to values of:

$$\frac{h_x}{h_y}\sqrt{\frac{\lambda_y}{\lambda_x}}$$

for which no charts are available, may be readily determined by interpolation.

It should be noted that the charts naturally enable the determination of fin efficiencies as generally calculated, viz. disregarding heat conductance of the fin in the flow direction and the warming up of the medium, since this way of determination may be regarded as the accurate fin efficiency related to the value of C = infinite.

Denoting the fin efficiency calculated in this way by ε_R, in conformity with the well-known formula:

$$\varepsilon_R = \frac{th(mh)_x}{(mh)_x}, \tag{7}$$

then, according to the charts given above, given the fin geometry (h_x length, h_y depth and v_0 thickness); the fin material (one having a heat conductance of λ_x perpendicular to the flow and λ_y in flow direction); the thermal capacity of the heat-transfer medium (C_k); and the heat-transfer coefficient (α), the accurate fin efficiency can be determined.

It appears from the charts that a given fin design will have different fin efficiencies corresponding to the different modes of operation. The points pertaining to these different modes of operation are plotted on the chart in a continuous curve.

With a given fin design, so that the geometry and material characteristics of the fins remain unchanged, the heat-transfer coefficient in most cases will be a continuous function of the mass rate of flow and with it, of the thermal capacity of the flowing medium:

$$h_x;\ h_y;\ v_0;\ \lambda \text{ constant, and } \alpha = f(C_k). \tag{8}$$

Consequently, in the accompanying efficiency charts, each fin design has its own characteristic curve for a given heat-transfer medium.

THE INTRODUCTION OF FIN EFFECTIVITY (ϕ_k)

There is no difficulty whatsoever in introducing a parameter related to fin efficiency. This parameter is generally defined for heat exchangers [5] and known as "effectiveness"—denoted by ϕ_h. (The value of ϕ_k we wish to introduce, is distinct from the ϕ_h defined for heat exchangers, in that our T_0 stands for the temperature of the fin base and not for the other heat-transfer medium. If the temperature of the fin base fairly approximates that of the medium flowing on the far side, the two values will coincide. This is the case, for instance, when the fin transfers heat to gas, while the fin base acquires heat from condensing steam or a liquid having a good heat-transfer coefficient.)

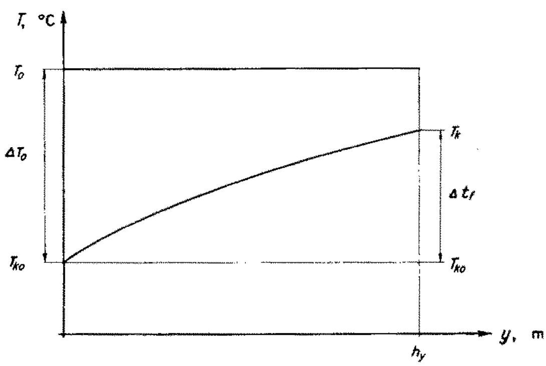

FIG. 7. Average temperatures of the medium and the fin base in the function of the distance from the point of inlet.

With the denotations of Fig. 7, it may be written as a definition that:

$$\phi_k = \frac{\Delta t_f}{\Delta T_0}. \tag{9}$$

It follows furthermore from the definition of the fin efficiency that:

$$Q = \varepsilon\alpha \,.\, F\Delta T_{\log}, \tag{10}$$

respectively, introducing the thermal capacity of the medium:

$$Q = \phi_k I \,.\, \Delta T_0. \tag{11}$$

Applying (11) to (10):

$$\phi_k = \varepsilon \frac{\alpha F}{I} \frac{\Delta T_{\log}}{\Delta T_0}. \tag{12}$$

However, in view of the relation between the mean temperature difference and the temperature difference at the inlet:

$$\frac{\Delta T_{\log}}{\Delta T_0} = \frac{1}{T_0} \frac{\Delta t_f}{\ln[\Delta T_0/(\Delta T_0 - \Delta t_f)]} = \frac{\phi_k}{\ln[1/(1-\phi_k)]}, \tag{13}$$

it may be written that

$$\frac{1}{C} = \frac{2\alpha h_y}{C_k} = \frac{\alpha 2 h_x h_y}{C_k h_x} = \frac{\alpha F_0}{I} \tag{14}$$

and

$$\frac{\varepsilon}{C} = \ln \frac{1}{1-\phi_k}. \tag{15}$$

The above will have proved that each ϕ_k value has a straight line in the $\varepsilon - C$ coordinate plane, intersecting the origin. These straight lines, in the more illustrative $\ln \varepsilon - \ln C$ diagram, transform into a series of parallel lines at 45°. The lines have not been plotted in our charts but they have been made readily demonstrable in a diagram with a scaled border.

Introducing the ϕ_k value and scaled border in the charts, orientation becomes considerably easier, a given temperature pattern having been related to each point.

Let us finally examine briefly, how the value of ε_L is related to our fin efficiency chart.

The definition of ε_L is as follows:

$$Q = \varepsilon_L \alpha F_0 \Delta T_0. \tag{16}$$

Applying (16) to (11) we may write that:

$$\varepsilon_L \alpha F_0 = \phi_k I. \tag{17}$$

Finally, taking into consideration also the (14) relationship, we may write that:

$$\varepsilon_L = \phi_k C, \tag{18}$$

whereby the simple correlation of the ϕ_k value of the fins, "fin effectiveness" and the value of ε_L in relation to the temperature difference at the inlet (similar to fin efficiency), have been illustrated.

DEFINITION OF THE FIN EFFICIENCY AND THE HEAT-TRANSFER COEFFICIENT OF FIN TYPE HEAT EXCHANGERS FOR THE EVALUATION OF EXPERIMENTS

In evaluating the measurements carried out on fin-type heat exchangers, the determination of the fin efficiency and the heat-transfer coefficient are fundamental requirements. In the approximate determination of the fin efficiency from equation (7), this is generally satisfied in such a way that the product of the heat-transfer coefficient and the fin efficiency is calculated from the measured thermal output of the heat exchanger, then the values of the heat-transfer coefficient and fin efficiency are determined through iteration, using formula (7).

Using the charts and the accurately determined fin efficiency, not only can the lengthy iteration procedure be avoided, but the actual value of the heat-transfer coefficients can be determined. In the processes followed so far the error of the fin efficiency calculations was transposed into the value of the heat-transfer coefficient, the measurement having yielded the product of these two.

To facilitate evaluation, let us introduce the following variable:

$$r = C(mh_x)^2 = \frac{C_k}{\lambda_x} \frac{h_x^2}{v_0 h_y} = \frac{I}{F_0} \frac{2h_x^2}{\lambda_x v_0}. \tag{19}$$

Its curves were indicated in the charts with staggered lines.

Now the characteristic values of the measured fins are easy to determine, since r may be computed from the known quantity of the heat-transfer medium and the geometric characteristics of the fin design, while the value of ϕ_k can be determined from the measured temperatures.

The two values so determined define one single point in our chart. This is the one characterizing the fin design under the given operation conditions. Measuring the temperatures under various operation methods so as to calculate ϕ_k, the points plotted in the chart will yield the aforementioned characteristic curve of the fin design. In its possession both the accurate heat-transfer coefficient and the accurate fin efficiency can be determined, from each point of the curve.

In spite of the fact that it takes account both of the heat conductance of the fin in the flow direction and the warming up of the flow medium, the evaluation of the fin design according to this method is considerably easier than the usual approximate evaluation according to formula (7). This is due to the fact that the computation starts from simply produced factors (r, ϕ_k), derived directly from measurements (temperatures, mass rates of flow) and yields the actual fin efficiency or the actual heat-transfer coefficient from the value of mh_x.

THE ERROR DUE TO NEGLECTING HEAT CONDUCTANCE IN FLOW DIRECTION AND THE WARMING UP OF THE MEDIUM

The significance and sphere of application of the calculation of the accurate fin efficiency may be determined by calculating the error of the value by means of formula (7), against the accurate fin efficiency as derived from the charts.

Taking the status of the fin to be $mh_x = 1{\cdot}5$, which is frequent in heat engineering, we have illustrated in Fig. 8, as the function of two specifically chosen parameters, the percentage error caused by the above approximation. The figure will clearly show that in several areas the magnitude of the error is considerable.

It can be observed that the magnitude of the error increases with the fin depth. This means that by larger fin depth the value of the fin efficiency can be substantially reduced in comparison to what is calculable with the (7) formula.

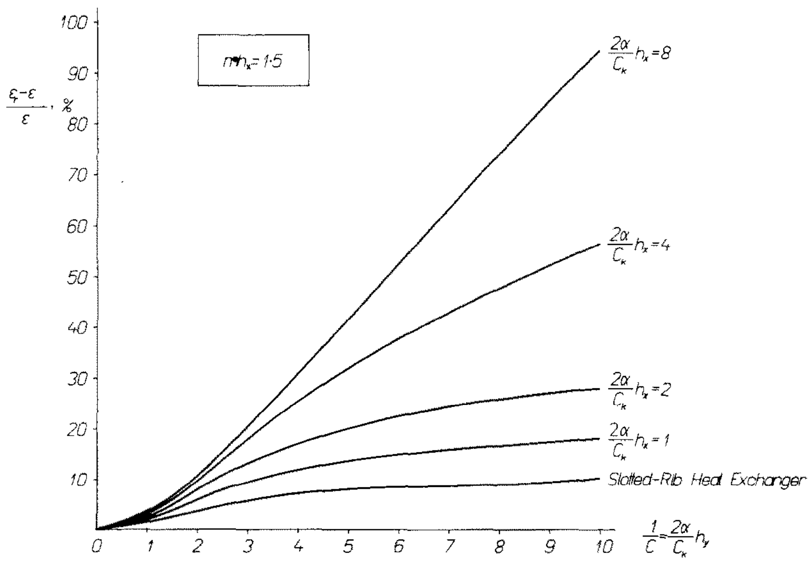

FIG. 8. Error due to neglecting the heat conductance in flow direction and the non-uniform changes in the temperature of the medium [at $(mh)_x = 1{\cdot}5$].

Let us consider furthermore that the heat-transfer coefficient tends to increase according to a power of the mass rate of flow, having an exponent below 1—which permits the conclusion to be drawn that the value of α/C_k generally increases with diminishing mass rates of flow—as shown in Fig. 8—the relative error will also increase.

All this is obvious qualitatively, since the ε_R fin efficiency calculable by the (7) formula is derived by means of an approximation, assuming the temperature of the flowing medium along the fin to be constant. This takes place with a zero depth of the heat exchanger, that is, with infinite thermal capacity of the flow medium.

THE EFFECT OF HEAT CONDUCTANCE ON FIN EFFICIENCY IN FLOW DIRECTION

Figure 1 shows the fin efficiencies (this case refers primarily to heat exchangers with slotted fins) with zero conductance in the direction of flow. It will be interesting to compare the curves of Fig. 1 with those in the other five, in which heat conductance in the direction of flow, and normal to it, coincide. The comparison shows that at given C and $(mh)_x$ values, the highest fin efficiency corresponds to the situation where the value of the heat conductance in flow direction is zero. This, in other words, means that the mere slotting of any kind of plate fins normal to the flow, even though it does not modify the heat-transfer coefficient, would cut heat conductance in flow direction, and increase the output of the heat exchanger.

This throws light on a special advantage of slotted fins, over and above the improvement of the heat-transfer coefficient.

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Résumé—On montre que l'on ne doit pas négliger les changements de température non-uniforme d'un milieu s'écoulant le long d'ailettes planes, changements provenant de différentes variations de température ayant lieu au voisinage de la base et de l'extrémité des ailettes et de la conductance thermique de ces ailettes dans la direction de l'écoulement, lorsqu'on dimensionne leurs surfaces.

On montre qu'avec de faibles débits massiques, une configuration d'ailettes efficace, ou de longues ailettes dans la direction de l'écoulement, comme par exemple avec des ailettes compactes ayant de trés bons coefficients de convection, des tubes avec des ailettes longitudinales dans une direction axiale, etc. . . ., une telle négligence peut conduire à des erreurs de 10 à 30 pour cent.

Les auteurs, ayant évalué les résultats finaux de leurs recherches grâce à un calculateur [2–4], présentent des diagrammes pour projets dans lesquels on a tenu compte des effets ci-dessus.

Les auteurs décrivent enfin un procédé basé sur leurs diagrammes, qui permet de calculer les résultats des mesures sur des surfaces avec ailettes plus simplement, plus rapidement et avec plus de précision que la formule habituelle d'efficacité des ailettes ne tenant pas compte des effets dont on a parlé.

Zusammenfassung—Es wird gezeigt, dass ungleichmässige Temperaturänderungen eines Mediums das entlang ebener Rippen strömt bei der Dimensionierung der berippten Fläche nicht vernächlässigt werden dürfen. Sie beruhen auf Temperaturänderungen nahe dem Rippenfuss und dem Rippenende und auf Wärmeleitung in der Rippe in Richtung der Strömung.

Es wird gezeigt, dass bei kleinem Durchsatz und wirkungsvoller Rippenauslegung, oder bei langen Rippen in Strömungsrichtung z.B. in Rippenbündeln mit sehr gutem Wärmeübergang oder längsgerippten Rohren, diese Vernachlässigung zu Fehlern von 10–30 Prozent führen kann.

Die Endergebnisse der Untersuchungen wurden mit Hilfe eines Computers ausgewertet [2–4]. Auslegungsidagramme, welche die erwähnten Einflüsse berücksichtigen sind wiedergegeben.

Schliesslich wird aufgrund der Diagramme ein Verfahren beschrieben wonach die Messergebnisse für berippte Flächen einfacher, schneller und genauer auszuwerten sind als mit der üblichen Gleichung des Rippenworkungsgrades, die obige Einflüsse vernachlässigt.

Аннотация—В статье указывается, что при определении размеров оребренной поверхности нельзя пренебрегать неравномерностью температурного режима среды при обтекании плоских ребер, возникающей в результате различных изменений температуры у основания ребра и на его вершине, и теплопроводностью ребра в направлении потока.

Показано, что при малых массовых расходах, эффективной конструкции или большой длине ребра в направлении течения (например, в случае компактных ребер, при продольном оребрении труб и т.д.) такое пренебрежение может привести к ошибкам до 10–30%.

С учетом вышеуказанного построены графики для определения размеров ребер на основании данных авторов [2, 3, 4], рассчитанных на вычислительной машине.

Наконец, показывается, как с помощью предложенных графиков можно обобщить результаты измерений на оребренных поверхностях проще, быстрее и точнее, чем с помощью обычной формулы полезной отдачи ребра.